

Investigating the Interaction Between Crowd Dynamics and Train Operations Through Agent-Based Modeling

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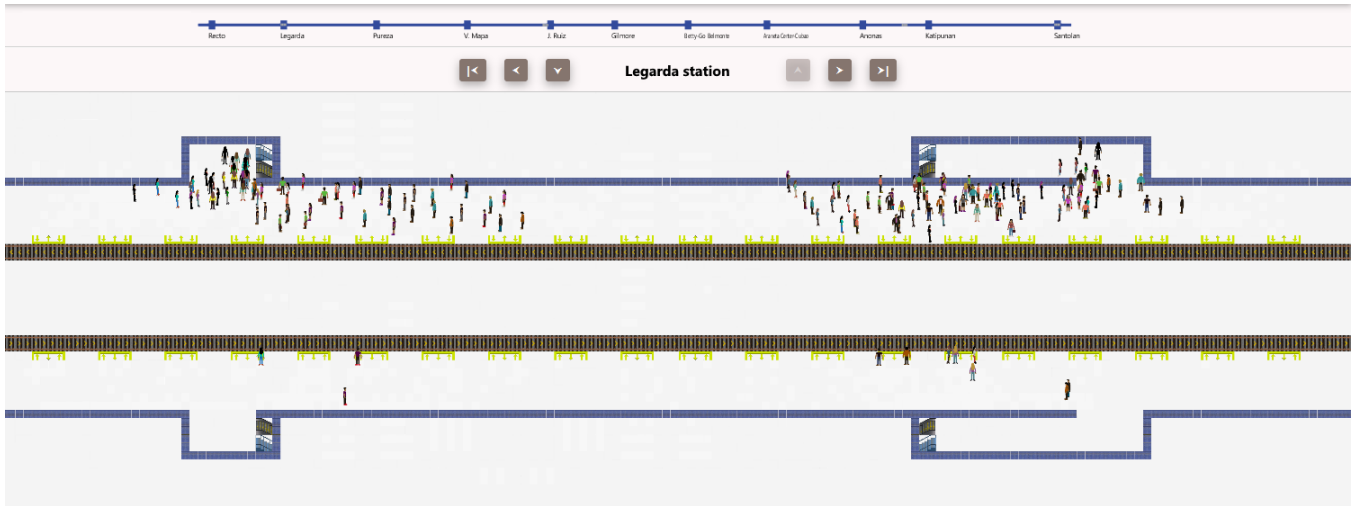


Figure 1: A screenshot of the visualization of the developed agent-based model showing passengers who just alighted from a train leaving the platform for the stairs in the simulation of the Legarda station of the LRT-2 line in Metro Manila, Philippines.

ABSTRACT

While agent-based models (ABM) have been developed for the planning of rapid transit systems (RTS), we have determined that the simultaneous modeling of their train operations with passenger crowd dynamics have not been adequate. We examine the interaction between train operations and passenger crowds in an RTS through an ABM which comprehensively integrates such train and crowd components, with the RTS of Metro Manila in the Philippines as a case study. After validating the model using video recordings of stations and smart card trip data, scenarios were tested on one of the RTS. It was observed that deploying less than 4 trains results in a considerable increase in the mean passenger travel time. When ridership is scaled alongside the 2015-2020 Philippine population growth rate of 1.63%, projected increases of 0.78% in the mean passenger travel time, 2.16% in the mean total time spent in the system, 4.7% in the mean train load factor, and 0.47% in the time trains stop at the platforms were observed between 2019 and 2023.

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CCS CONCEPTS

• **Computing methodologies** → **Agent / discrete models; Model verification and validation; Modeling methodologies; Simulation evaluation;** • **Applied computing** → Transportation.

KEYWORDS

agent-based model, rapid transit systems, trains, crowd dynamics

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1 INTRODUCTION

Rapid transit systems (RTS) are a mode of urban transportation, usually on an exclusive right of way, which carry large volumes of passengers between designated transit stations [6]. Oftentimes, transportation planners construct RTS to support the increasing population density in cities. However, the said increases in population density require that the complexities of the RTS associated with such urban centers evolve as well [19]. [9] underscore the importance of understanding how passengers use public transport in the operation and planning for the underlying transportation networks. Various methods have been devised for and used by transportation planners to gain valuable insights and plan strategies for the

management of RTS despite their increasingly complicated nature. Capturing the complex behavior of such systems is most appropriately done through another simulation modeling paradigm, called *agent-based modeling* [3].

In the context of agent-based modeling of rail-based transportation systems, it is often the case that the trains of the system at least partly constitute the agents in the agent-based model (ABM). There are also ABMs that have exclusively been made about crowd dynamics in the context of RTS environments, such as in [1]. In these models, because the emphasis is on the crowd behavior, it is each member of the crowd which mainly constitutes the agents of the ABM, so the trains need not be directly modeled. Using ABMs for these contexts is desirable because the behavior of crowds are recognized to be caused by the interaction of its independent pedestrians and their surrounding environment, making it a complex system [20]. Finally, there are models that incorporate both train agents with commuter agents, such as in [22]. In the studies where passenger dynamics are modeled alongside train movement, the incorporation of passenger crowd dynamics is limited and often use more analytical methods as opposed to agent-based means. Hence, it is imperative that an ABM intent on capturing emergent behavior from such RTS network should simultaneously consider the facts that (a) the trains, with their collective behavior within an RTS, are complex systems [13], and that (b) the passengers, with their collective behavior within an RTS, are also complex systems [20].

[17] developed an ABM of the transportation networks specific to their region as it was noted that current models were not suitable for their place. In a similar vein, [19] posed that it should be possible to adapt the methods used in the construction of their Singaporean RTS model to other RTS deployments in other cities provided that necessary data, such as that of infrastructure and travel demands, are available. Hence, it follows that it is admissible to adapt the approaches used in other transportation contexts to local conditions because of the presence of characteristics that are unique to a region, such as unorganized road behavior [17], or the penultimate station effect [19]. In the case of the National Capital Region (NCR) in the Philippines, also known as Metro Manila, it is natural to develop an ABM that integrates train operations with passenger crowd dynamics to capture emergent behavior that could only arise from attributes and qualities specific to the train system networks in the metropolis. Having said these, in order to faithfully capture the emergent behavior affecting the local RTS, both the behavior of the trains and the dynamics of the passengers as a crowd should simultaneously be modeled, while taking local factors into consideration, to investigate how train movement and passenger crowd dynamics affect the efficiency of the RTS networks in Metro Manila.

2 RELATED AGENT-BASED MODELS OF TRAIN SYSTEMS

The discussion on the agent-based models of rapid transit systems deemed most related to this work were classified into four parts. The first part covers general agent-based approaches in transportation studies that do not neatly fit into the succeeding categories. The second part covers transportation studies in which trains are the primary agents present in the model, without any presence of

passenger crowds. The third part covers transportation studies which include the presence of crowds and their dynamics (e.g., movement) in the model, but excludes direct representation of trains. Finally, the fourth part covers transportation studies which integrate both train and crowd agents in the model. Special attention to these papers were given, as the integration of both such agents in line with the goals of this research.

2.1 General Applications in Train System Modeling

[13] described a plan for the simulation of the Netherlands Railways with the intention of finding links between travel supply and demand by evaluating the effects of pricing strategies on passenger behavior and on the financial and operational aspects of the overall network. [13] argued for the need to build a simulation model for capturing the complex dynamics of railway networks with passengers because of the coupled nature of the system components and the nonlinearity of its performance metrics.

[17] argued that several existing transportation models are not suitable for developing countries because their transportation networks are not always organized. In view of such argument, [17] introduced an ABM for transportation network simulation incorporating unorganized traffic behavior, using the road networks of the city of Hanoi in Vietnam as a case study. The organization of the road network is based on Geographic Information System (GIS) data, as with the work by [5].

2.2 Models Solely of Train Operations

The studies wherein the models utilize train agents without passenger components tend to focus on freight transport. For instance, [7] presented a simulation to model the flow of intermodal terminal units (ITU) between terminals to simulate the integration of rail-based and road-based transportation of cargo. Train agents represent the rail-based vehicles, while truck agents represent the road-based vehicles. Train terminals, which serve as the liaison between the train and truck vehicles, were also modeled as agents. The agent-based model was made in the form of a discrete event simulation (DES) model using MODSIM III, a simulation software.

Likewise, [5] presented an ABM to evaluate the profitability of transshipment technologies from the perspective of rail freight companies. The model, similar to that of [7], also simulates the movement of cargo trailers between multiple destinations using train agents, among others. The train agents were assigned parameters governing their behavior, such as the cargo capacity of each agent. In validating the model, the real-world schedules of the trains were implemented and simulated.

2.3 Models of Crowds in a Transportation Context

[20] proposed a hierarchical approach to the simulation of crowds and their dynamics by integrating factors such as agent perception and cognitive control to reproduce self-emergent phenomena observed in real life. As a case study, [20] modeled and analyzed the entrance of the Xi'an railway station in China using the ABM.

Likewise, [1] focused on illustrating and developing agent-based passenger motion and behavior model for applications in public

transport. The model was designed to fit the needs of public transportation instead of the more general pedestrian models in other studies. [1] noted that crowd simulation may be done in two different ways: through *macro models*, which represent the crowd as a malleable mass with fluid-like behavior; or through *micro models*, which try to individually capture the movement of each member in the crowd, relying on emergent crowd behavior. Emergent behavior such as the formation of lanes (similar to the observation of [20]) and the asymmetric formation of crowds on bus doors were observed. [4] built an ABM that incorporates group cohesion forces to crowd egress scenarios in movement-restricted public spaces to observe whether groups within the crowd have an overall effect in the crowd's movement.

Meanwhile, [25] predicted the effects of train station closures of the Beijing rail transit system in China on commuter decisions (at the microscopic level) as well as travel demand (at the macroscopic level). [25] recognized two methods in predicting passenger behavior: the *data-driven* approach, wherein recognition and prediction of patterns of passenger behavior are based on *existing* data; and the *model-driven* approach, wherein a qualitative analysis incorporating passenger behavior are built and validated against available real-world data. In the case of [25], a combination of smart card data and passenger surveys were used to validate the models.

Finally, [21] developed an agent-based crowd dynamics model of commuters in one of the high-volume terminal stations of the MRT-3 line in Metro Manila in the Philippines. The model was made with the intention of discovering emergent behavior in crowd formation in order to assess infrastructure preparedness. [21] noted that the physical layout of the station, along with how the passengers within it behave, contribute to the congestion of the station. However, [21] noted that the model does not consider the heterogeneity of passenger attributes and behavior which may be factors that affect crowd dynamics.

2.4 Integrated Models of Train Operations and Crowd Dynamics

The work by [11] deals with the structural aspects of modeling of interactions between mobile entities (e.g., vehicles, passengers) in ABMs of transportation terminals. Their study proposes a dedicated managing agent responsible for the handling the interactions between mobile agents in the simulation. The train agents, which move on line-based infrastructure, are controlled by managing agents as non-intelligent simple entities.

In an extension of the work by [12], [19] integrated a full-scale (composing of all seven operational lines at that time, as opposed to only of a single train line) ABM of Singapore's RTS with mathematical models of route choice and information from smart card data to provide a more comprehensive understanding of crowd dynamics as opposed to analytical models. The model also incorporates station-specific walk times measured from field visits, as it was recognized that travel times include a walking component. Using the developed simulation, population scaling scenarios in Singapore were performed, and it was observed that there was a tipping point with regard to the population with respect to the capacity of the RTS wherein the quality of service significantly degrades after that point is passed.

The research by [2] aims to eliminate delays in passenger reactions when waiting in bus rapid transit (BRT) stations by designing an improved passenger information system (PIS). This was facilitated by the creation of an ABM of a BRT station in the city of Brisbane in Australia. Its input parameters were calibrated using smart card data, field measurements, and video recordings (similar to what was done by [21]). An ABM was built using AnyLogic to describe bus arrivals and departures, as well as detailed passenger movement.

An ABM for the Philippine MRT-3 train line was developed by [22] using the NetLogo ABM environment. Like in the study by [12], and complementary to the crowd-focused study by [21], three agents were identified: train, commuter, and station agents. The modeled infrastructure contains a depot where trains spawn, as is in the real world. The model allows the user to run multiple operational scenarios.

Lastly, [14] presented an ABM of a passenger rail system using an activity-based simulation approach to predict the impact of pricing strategies. The population and temporal flexibility of the passenger agents were set in accordance with a passenger survey dataset, as with the previous work by [25]. The simulation was created in MATSim, a framework for implementing agent-based transport simulations, and was scaled down to 10% of the reference train system.

3 METHODOLOGY

The processes of this study were divided into three phases:

- (1) *Design and development*, concerned with the formulation and creation of the pertinent systems and models,
- (2) *Validation and calibration*, concerned with the comparison of the model with empirical data to measure its fidelity to the real-world systems, as well as the changes that are applied to rectify perceived inaccuracies between simulation and real-world behavior, and
- (3) *Experimentation and analysis*, concerned with the execution of the model on different scenarios and the investigation of their results.

3.1 Design and Development

This phase may be further divided into three sub-phases: the development of a simulation of the train operations, the development of a crowd dynamics model, and the integration of the latter two components. In the development, the Java programming language was used, while the SQLite relational database management system (RDBMS) was utilized for organizing the infrastructure data for the train simulation. The outputs of this phase were the two systems as described in Section 5; namely the train simulation, and the station editor.

3.2 Validation and Calibration

Two classes of empirical data were prepared for use in the validation: the smart card trip data, and the video recordings of the train stations. The smart card trip data contains turnstile tap-in and tap-out data that were used to quantitatively validate the simulation results, while the video recordings of the train stations were taken by the researcher on-site in the important areas of some stations in

one of the modeled RTS, namely, the LRT-2 line. These station video data were used to qualitatively validate the simulation behavior, specifically those of the passenger crowds. As for the quantitative validation, the model was run, with its results compared with those of the empirical data. Using the results acquired in a round of simulation, the model was then revised to address perceived issues and inaccuracies. It should be noted that the results of the quantitative validation were also analyzed in the succeeding phase as described in the next section. That is, the act of validating the model quantitatively already doubles as an experimentation process. Hence, this phase overlaps with the next one, and need not necessarily be seen as distinct. See Section 6 for the specifics of this validation process.

3.3 Experimentation and Analysis

As mentioned previously, this phase overlaps with the previous one in that an experimentation process was already performed when validation was done. The results acquired from the simulation were analyzed, and observations and trends were pointed out. See Section 6 for these analyses. The tools and frameworks used for the analyses were Python with the NumPy, SciPy, and Dask libraries, pivot tables using Microsoft Excel, and further statistical tests using MATLAB. In addition to the analysis of the experimentation results resulting from the validation processes, scenario tests were also performed, and their results likewise analyzed. Section 7 describes these in detail.

4 COMPONENTS OF THE MODEL

In agent-based modeling, a system is modeled in terms of autonomous decision-making entities called *agents*, the *environment* these agents are on, and the interactions within and between such components. Following from this, the discussion of each component of the developed ABM of the RTS in this research is divided into the two such categories. Moreover, the ABM developed for this work is hereafter referred to as the *train simulation*.

4.1 Agents

The two salient agents in the train simulation are the *train agents*, and the *passenger agents*. The design and behavior of both are discussed below.

4.1.1 Train Agent. In RTS, trains are the vehicles which carry passengers between stations. Trains spawn and despawn at a depot (further described in Section 4.2.1). Trains move along tracks and stop regularly at stations to allow passengers to board and alight it. Each train maintains a *station queue* containing the stations where it should stop in order. Once a train exhausts all the stations in its station queue, this queue is replenished, but in reverse, signifying that the train should now go in the other direction. The trains use the Dijkstra's algorithm to find the shortest path to its next station. A flowchart detailing the general operations of train agents is shown in Figure 2. Each train is composed of multiple carriages. Each carriage has a set passenger capacity. The locations of passengers are not modeled inside of the train carriages.

4.1.2 Passenger Agent. Passengers agents use the train system and ride its trains in order to get to a destination station from a station of

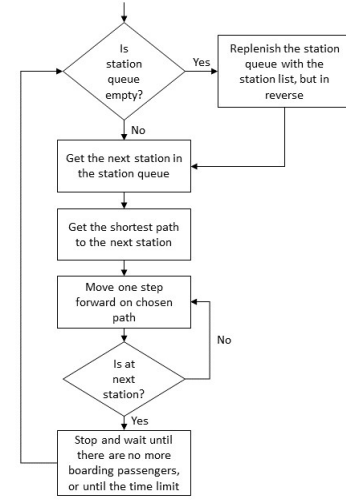


Figure 2: A flowchart showing the general operations of train agents in the train simulation.

origin. A flowchart detailing the general operations of train agents is shown in Figure 3.

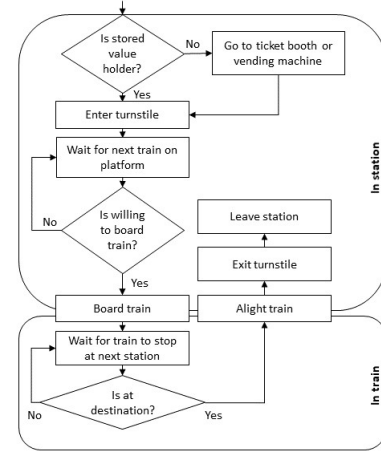


Figure 3: A flowchart detailing the general decision-making processes of passenger agents in the train simulation.

Three status variables generally describe the cognition of each passenger agent, in order of decreasing granularity:

- (1) The *disposition*, representing whether the passenger is going to ride a train, riding a train, or going to exit a station,
- (2) The *state*, signifying the general location of the passenger with respect to the type of its goal (or lack thereof), and
- (3) The *action*, denoting the specific activity of the passenger.

Likewise, three variables primarily constitute the spatial parameters of a passenger agent in most cases:

- The *position*, denoting the two-dimensional coordinates of the passenger agent on the environment it is on,

- The *heading*, denoting the direction the passenger agent is facing, and
- The *walking speed*, denoting the distance the passenger agent walks per simulation tick.

At the microscopic level, the movement of a passenger agent when walking freely in the train simulation is governed by behaviors based on those defined in the social force model for pedestrian movement by [8] (Figure 4). These behaviors are:

- (1) That the social force must describe an acceleration towards the desired velocity of motion,
- (2) That the social force must observe that distance from other pedestrians and borders in the environment are maintained, and
- (3) That the social force must model attractive effects by other pedestrians or objects.

When queueing for a certain goal (e.g., a ticket booth), the movement of a passenger is also influenced by static floor fields predefined in the environment that lead to such goal. Static floor fields are elaborated further in Section 4.2.2.

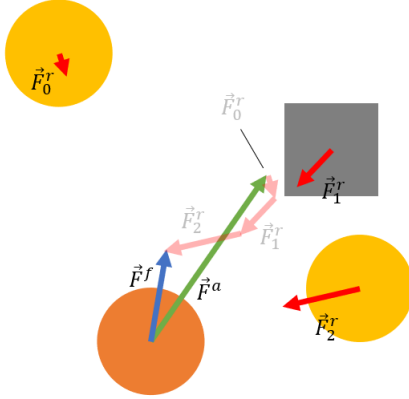


Figure 4: An illustration of how passenger agents in the train simulation move based on behaviors laid out in the social force model by [8]. In the figure, the desired movement vector of the orange passenger agent (represented by \vec{F}^a) is affected by repulsive forces from an obstacle in gray and the other passenger agents in gold (represented by \vec{F}_0^r , \vec{F}_1^r , and \vec{F}_2^r). The resultant motivational vector of the passenger is represented by $\vec{F}^f = \vec{F}^a + \sum_{k=0}^{n-1} \vec{F}_k^r$, where n is the number of repulsive forces (in this case, 3).

Each passenger agent may hold two types of tickets: a *single journey* (SJ) ticket, or a *stored value* (SV) ticket. SJ tickets are cards which are issued to passengers at their origin station, then returned by the passengers at their exit station. On the other hand, SV tickets are cards which passengers may keep with them even after exiting the RTS. The type of ticket the passenger agent holds also determines the cognition of the passenger agent. Specifically, in the train simulation, SJ holders are assumed to be passengers who are new to the system, and hence unfamiliar with the environment of the train stations. Given this, when SJ holders encounter obstacles they get stuck on, they will find a path around it to free themselves. In

contrast, SV holders are assumed to be regular commuters, and hence, are more familiar with the station environments. Therefore, SV holders perform pathfinding beforehand. In short, when speaking of how passenger agents generally avoid obstacles, SJ holders are *reactive*, while SV holders are *proactive*. It should be noted that such pathfinding processes are necessary in addition to the application of the social force behaviors as mentioned above as their sole use do not guarantee that a passenger agent will get past a set of obstacles. The A* search algorithm is used for the pathfinding of the passenger agents.

Aside from the attributes mentioned above, each passenger agent also possesses other properties such as the gender of the passenger, as well as the demographics it belongs to. The gender of the passenger is considered in certain gender-restrictive facilities in the Metro Manila RTS, such as in female-only train carriages. The latter, meanwhile, determines the walking speed of the passenger, using data from the Philippine demographics in [10] combined with the study on walking speed measurements per demographic in [23].

4.2 Environments

The environments in the train simulation may be classified into the *train line infrastructure*, which is an environment solely used by the train agents, and the *train station*, which are used both by passenger and train agents.

4.2.1 Train Line Infrastructure. In the train simulation, the train line infrastructure is a network of train tracks that constitute the *mainline* and the *depot* of an RTS. The mainline is made up of the train tracks that connect the train stations of the RTS, representing the tracks used primarily for passenger operations. Meanwhile, the depot consists of the actual structure where trains enter and exit the system, as well as the *spur* train tracks which connect such structure to the mainline. In this train simulation, the internal track network of the depots are abstracted.

4.2.2 Train Station. Stations are structures in a train line where trains regularly stop to load and unload passengers. Each train station in the train simulation may be composed of one or more floors. Each floor is represented as a grid of square floor units called *patches*. A patch is the basic unit of the station floor environment. Each patch has an area of 0.6 m^2 , based on the approximate area occupied by an average Filipino in the measurements by [18].

Aside from examining stations in terms of their structure, the parts of a station environment may also be classified according to their utility to the passengers. For instance, the part of a station which accepts passengers from entrances and which contains the ticket booths is called the *concourse* of the station, while the parts of the station which allow passengers to wait for, board, or alight trains at the side of the tracks are called *platforms*. Each of these two parts are characterized by the presence of certain *amenities*, which are objects in the train station which perform certain functions and services for the passengers. Table 1 lists all possible amenities that may be added in a train station in the train simulation.

Static floor fields are used to facilitate passenger movement when queueing for a certain goal. Each goal that supports queueing, called a *queueable*, maintains a list of patches associated with its static floor field. Each of the patch in this list is associated with a value

Table 1: The amenities that may be added in a train station in the train simulation. Amenities with asterisks are required in all train stations, while the rest are optional.

Amenity	Description
Station entrance/exit*	The entry and/or exit points of the station, where passengers spawn or despawn.
Security*	Where passengers are checked by security personnel before entering the concourse.
Ticket booth*	Machines where train tickets are dispensed.
Turnstile*	Facilitates the use of tickets for passengers to be allowed access to the platforms.
Train boarding area*	Markings on the platforms where the train doors should align, designating where passengers should wait.
Stairs	Vertical walkways composed of steps which allow passengers to go to another floor.
Escalator	A moving staircase which carries passengers to another floor.
Elevator	A machine that vertically transports passengers from one floor to another.
Obstacles	Objects on the station which serve as boundaries, barriers, or demarcations for passengers.

of the range $(0.0, 1.0]$. These values denote the likelihood of the passenger agents in selecting the patch in question when queueing for the goal. Intuitively, floor fields represent a heat map of the queue that forms for a specific goal. The use of static floor fields in this work is similar to that of [21].

5 SYSTEM DESIGN

Two systems were developed for this research: the *train simulation*, which is most related to the main output of this study, and the *station editor*, which is a companion system for the train simulation.

5.1 Train Simulation

The train simulation is the primary system developed for this research to be used in running experiments. It consists of a simulation of a single RTS with all its stations and the tracks connecting them, with their distances to scale, in accordance with the real-world measurements of the line. The train simulation derives its data from three external data sources:

- The *infrastructure data*, contained in a relational database, which contains the necessary dimensions of the train line infrastructures as well as the rolling stock (train fleet) data and their parameters,
- The *station spatial layout data*, contained in .stn binary files, containing the actual spatial layout of the stations with all its floors and the amenities contained therein, and

Table 2: The output logs maintained and saved by the train simulation.

Type	Description	Updated Every
Passenger log	Contains all completed passenger trips and related data.	Every time a passenger despawns from the model after completing its trip.
Station log	Contains station-related parameters at the current simulation state	Every one minute since the simulation started.
Train log	Contains train-related parameters at the current simulation state.	Every time the train leaves a station.

- The *smart card data*, contained in a .csv file, containing real-world smart card trips to be used in generating passenger agents in the train simulation.

For purposes of optimization, when the train simulation is run, each of the train stations are executed in parallel with each other, as the passenger agents and their dynamics within each station have no influence whatsoever on the other stations. Similarly, each floor within a train station may also be simulated in parallel with each other, as passenger agents who are within each floor do not affect the ones at the other floors.

The results of each round of simulation are stored in .csv log files. Table 2 describes each output log maintained and produced by the simulation.

5.2 Station Editor

The station editor is a companion system developed alongside the train simulation. It serves two purposes: to draw and validate the layouts of each train station to be used by the train simulation (which the station editor saves in a .stn binary file), and to serve as a smaller-scale ABM only for passenger crowd dynamics in a single train station.

6 VALIDATION AND ANALYSIS OF THE MODEL

Validation is the process of ensuring that there is a correspondence between the implemented model and its subject [24]. The validation of the train simulator was divided into two components: qualitative validation, which visually validates the movement of the passenger crowd using the station video recordings, and quantitative validation, which validates the simulation through comparison of its results against the smart card data.

6.1 Face Validation of Crowd Dynamics

The following behaviors were sought in the developed ABM, based on behaviors suggested by [16] to be observed when validating the crowd dynamics of agent-based models:

- (1) *Corner hugging*, described as when pedestrians slow down and "wrap around" corners,

- (2) *Lane formation*, described as when pedestrians who move in two opposite directions self-organize to form a lane for each direction, and
- (3) *Counterflow behavior*, described as when pedestrians coming in from different directions self-organize to avoid each other. This is similar to lane formation, but is more complex as passengers come in from multiple directions.

All these three behaviors were observed at all locations in all the simulated stations which were chosen to be observed. Figure 5 shows examples of these three key behaviors as observed in the ABM. In the first image from the left, passengers leaving the station are observed rounding the corner tight. In the second image, passengers of opposing directions self-organize their own lanes (lane formation), in order to avoid each other. In the last image, the passengers seen are from four different origins, yet still organize directions which are seen to avoid each other (counterflow behavior).

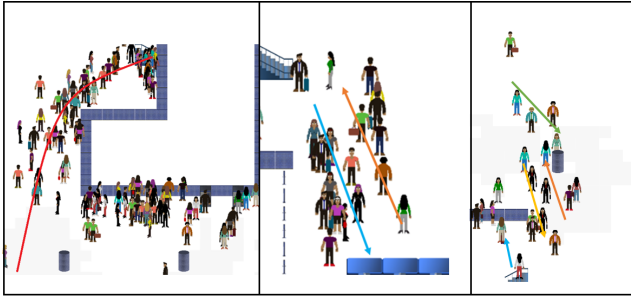


Figure 5: The three behaviors suggested by [16] to be observed when validating crowd dynamics models as seen in the train simulation. From left to right: corner hugging, lane formation, and counterflow behavior.

Aside from these, the behaviors of queueing at the concourses as well as most platform behaviors seen in the video recordings were also seen in the model. However, certain behaviors such as cohesion due to groups between passengers, moving closer to the train even before the train stops at the platform, expediting when the train doors are about to close, and a general sense of foresight when avoiding obstacles were not seen in the simulation, despite being observed in the video recordings. Moreover, some behaviors, such as the formation of wave-like hordes when exiting crowded staircases, and spending a prolonged amount of time being stuck on obstacles were only seen in the simulation, and not in the recordings.

6.2 Validation of the Model Against Smart Card Data

To measure how faithful the simulation results are to their real-world subjects, and can therefore produce credible results when doing experiments with them, smart card data was used as a reference to quantitatively measure the results of the train simulation. Each of the three integrated train system models was executed based on granular smart card data of trips on the 28th of January in the year 2019. Noting that each row of granular data represents

Table 3: The parameters for the simulation of the LRT-1, LRT-2, and MRT-3 train lines. These parameters were taken from social media announcements of the respective operators of each RTS closest to the simulated date of January 28th, 2019.

Train System	First Trip	Interval (min)	# of Trains to Deploy	Speed (km/h)	Carriage classes
LRT-1	4:30 AM	6	16	40	3G, 2G
LRT-2	5:00 AM	10	5	60	1G
MRT-3	5:00 AM	7	15	30	1G, 2G

a completed (tapped in, then out of a turnstile) trip from a passenger's origin to its destination station, the same rows were fed into each integrated train model. The differences in the real-world travel times (the time taken between tapping in and tapping out) and the predicted travel times by the simulation for the trips represented by such rows were then noted. For each RTS model (namely the LRT-1, LRT-2, and MRT-3), the simulation was run from 4:00 to 9:00 AM with the parameters as listed in Table 3.

6.2.1 First Round of Simulations (Iteration 1). The LRT-1 simulation has not been able to complete the entire 5 hour simulation from 4:00 to 9:00 AM as the memory requirements of the simulation exceeded those of the machine the simulator is running on. Hence, the simulation of the LRT-1 are limited to until 5:45 AM, which is around the time when the simulator halts due to the lack of memory. While the empirical data contained 19,411 rows of trip information, the simulation was only able to successfully track the completed trips of 2,918 passengers (15% of the empirical trips). Upon investigation, the uncompleted trips were caused either by (a) a passenger being in an unrecoverable inconsistent state within the station, and hence unable to complete its trip, or (b) a passenger being unable to be tracked by the passenger tracker due to concurrency and memory issues.

As for the LRT-2 model, 54,564 rows of the empirical trip data were used to spawn the passengers at the required time. The simulation was able to successfully track the completed trips of 30,002 passengers (55% of the empirical trips). As for the low trip completion rate, reasons similar to those seen in the LRT-1 model were identified.

Finally, for the MRT-3 model, 74,444 rows of the empirical trip data were used for passenger generation. The simulation was able to successfully track the completed trips of 26,955 passengers (36% of the empirical trips). As for the low trip completion rate, reasons similar to those seen in the LRT-1 and LRT-2 models were identified.

Summaries of the results after the first round of simulations of the LRT-1, LRT-2, and MRT-3 models are seen in Table 4.

6.2.2 Fixing the Station Entrance Backlogs Issue (Iteration 2). Investigations were made to further determine why a large portion of the passenger trips were not completed in the previous iteration, especially in the cases of the LRT-2 and the MRT-3. After also resolving the issue of passenger tracking, it was discovered that an unusually large volume of passengers were queueing outside the station gates. It was determined that the queues of the passengers to

Table 4: A summary of the statistics of each RTS model after the first iteration of validation, based on the differences between the simulation and the empirical travel times. A positive mean or median signifies an underestimation of real-world times by the simulation, while a negative mean or median denotes an overestimation of such times on the part of the model. The asterisk indicates that the LRT-1 model did not complete the validation process over the prescribed times of 4:00 to 9:00 AM.

Train System	Mean (s)	Standard deviation (s)	Median (s)	RMSE	NRMSE
LRT-1*	200.47	460.15	61.14	501.93	15.78%
LRT-2	99.94	142.65	91.06	174.18	20.10%
MRT-3	-469.66	397.69	-332.91	615.42	76.00%

Table 5: A summary of the statistics of each RTS model after the second iteration of validation, based on the differences between the simulation and the empirical travel times. A positive mean or median signifies an underestimation of real-world times by the simulation, while a negative mean or median denotes an overestimation of such times on the part of the model. The asterisk indicates that the LRT-1 model did not complete the validation process over the prescribed times of 4:00 to 9:00 AM.

Train System	Mean (s)	Standard deviation (s)	Median (s)	RMSE	NRMSE
LRT-1*	151.45	441.80	33.39	467.03	15.47%
LRT-2	-3.36	141.12	11.52	141.16	17.43%
MRT-3	-384.30	319.11	-290.83	499.52	35.16%

Table 6: A summary of the statistics of each RTS model after the third iteration of validation, based on the differences between the simulation and the empirical travel times. A positive mean or median signifies an underestimation of real-world times by the simulation, while a negative mean or median denotes an overestimation of such times on the part of the model. The asterisk indicates that the LRT-1 model did not complete the validation process over the prescribed times of 4:00 to 9:00 AM.

Train System	Mean (s)	Standard deviation (s)	Median (s)	RMSE	NRMSE
LRT-1*	171.97	605.06	-15.87	629.02	17.51%
LRT-2	-57.43	149.61	-47.34	160.26	19.78%
MRT-3	-434.40	340.95	-342.50	552.22	33.84%

some station's security gates blocked the station entrances, and that these queues were not observed to dissipate or resolve themselves over time. Figure 6 shows an example of this clogging phenomenon. Such issues were mostly resolved through making station layout adjustments.

After fixing the entrance backlogs issues as described previously, the models were run once again, yielding the updated statistics as shown in Table 5.

When comparing against the original versions of the models, all models of each train system in this iteration recorded improvements in terms of the root mean squared error (RMSE) metrics. Furthermore, with regard to the entrance backlogs issue discovered in the previous iteration, all models in this iteration saw a decline in the volume of entrance backlogs. However, the LRT-1 was still only able to run until 5:45 AM.

6.2.3 Changing How Trains Wait at Platforms (Iteration 3). Since the first version of the model, the trains spend a fixed time of 30 seconds in each platform to allow passengers to board and alight. The resulting models only represent a one-way integration of crowd behavior and train operations, where the latter influences the former. With the goal of two-way integration of crowd behavior and



Figure 6: Passengers queuing for the security gates were observed to block the station entrances in some stations, causing an unusually high volume of passengers queuing outside.

train operations in mind, amendments were made to ensure that the behavior of the trains are also affected by the passenger crowds. In the third iteration, the train stops at the platform for a minimum

Table 7: The mean, standard deviation, and median travel times (in minutes) that result when the LRT-2 simulation is run with the specified number of trains deployed.

	Number of trains deployed							
	1	2	3	4	5	6	7	8
Mean travel time (min)	49.61	35.17	26.93	19.74	18.49	17.68	16.96	16.16
Standard deviation (min)	28.90	18.79	13.65	7.87	7.59	7.50	7.46	7.16
Median travel time (min)	43.92	32.38	24.58	19.03	17.83	16.97	16.22	15.47

of 11 seconds and a maximum of 30 seconds, determined by observations on the station video feed where minimum and maximum waiting times of 11.14 and 30.3 seconds were observed respectively. The waiting time of the train is modeled using a linear function $w(n_p)$ as described below, where n_p is the number of passengers waiting at the platform where the train arrived:

$$w(n_p) = \begin{cases} 0.03832 \cdot n_p + 11.14 & n_p \leq 500 \\ 30.3 & n_p > 500 \end{cases}$$

The rounded value of $w(n_p)$ gives the waiting time of the train for its current platform in seconds. This waiting time shall be pegged to 30 seconds if there are more than 500 passengers waiting for the train at the platform.

Summaries of the results after the third round of simulations of the LRT-1, LRT-2, and MRT-3 models are seen in Table 6.

6.2.4 Summary of the Simulation Results Through All Iterations. Using the normalized root mean squared error (NRMSE) metric to compare the results of the latest iteration (Iteration 3) of each train system model with each other, it is noted that the LRT-1 model has the lowest value of such metric at 17.51%, followed by the LRT-2 model at 19.78%, and then the MRT-3 model at 33.84%. However, the fact that the LRT-1 validation was not able to be completed in the same time period as the other ABMs is once again raised, and that validation of equal rigor to the other models is urged in order to gain more confidence on the results of the LRT-1 simulation. Having said this, among the models which were able to complete the necessary time period of 4:00 to 9:00 AM, the LRT-2 model had the better NRMSE of 19.78%. The same model has also seen the best mean travel time difference at just under one minute of overestimation (-57.43 seconds). Hence, the model which best captures its respective train system with reasonable confidence was determined to be the LRT-2 simulation.

It should be noted that for the LRT-2, a worse RMSE was actually yielded by the third iteration. This signifies that the estimations of the train platform waiting time by $w(n_p)$ are far from the actual waiting times taken by the trains in real-world scenarios. This means that $w(n_p)$ should be refined in future works. Nevertheless, this general behavior of the trains stopping longer for a larger volume of passengers waiting at the platform is desired, and hence should be maintained.

7 SCENARIO TESTING

Having been determined that the LRT-2 model captures its real-world train system the best, two hypothetical scenarios were then tested with the said model. The first scenario inspected how the

passenger travel times change when more or less are deployed in the system. The least number of trains deployed that does not considerably worsen passenger journey times were then sought. Meanwhile, the second scenario analyzed how an increase in ridership brought about by population growth affects the operations of the LRT-2.

7.1 Changing the Number of Trains Deployed

The simulation was run with the same parameters as listed in Table 3, but with different numbers of trains deployed. Specifically, scenarios were run to see the consequences when 1, 2, 3, 4, 6, 7, and 8 trains (as opposed to just the standard 5 trains) were deployed. Table 7 shows the comparison of the mean travel times in the train system simulation given the amount of trains deployed, along with other metrics. In Figure 7, a graph of the average and median passenger travel time compared to the number of trains deployed in the simulation.

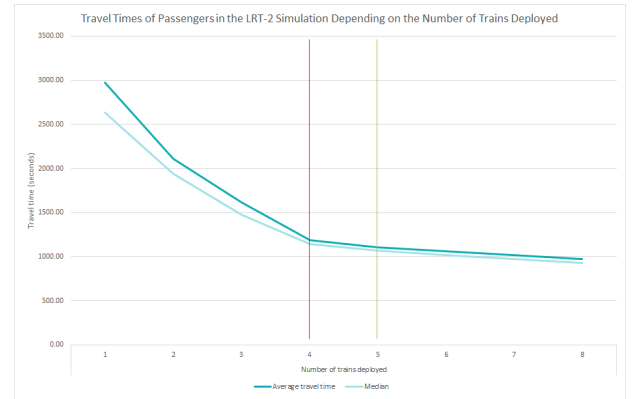


Figure 7: The average and median time spent by the passengers in the LRT-2 across different numbers of trains deployed. The green line indicates the standard number of trains deployed by the LRT-2 under normal operations (5 trains), while the red line indicates the number of deployed trains (4 trains) below which travel time metrics increase significantly.

Given these observations, it is concluded that while any change in the number of trains deployed will lead to a statistically significant change in the average travel time (as per the Wilcoxon rank sum test), deploying less than four trains will result in large increases in the passenger travel time, relative to what the changes in travel time would have been when deploying four or more trains.

Table 8: The projected LRT-2 ridership from 4:00 to 9:00 AM for a given year ahead of 2019 using the mentioned population growth rate of 1.63%. For each year projected ahead, a value of 1.63% of the previous year's trip counts is added back to that previous year's count. The first row (Year 0) contains the empirical smart card trip count from the year 2019, while the rest of the years contain projected counts.

Years projected ahead	Year	Projected trips from 4 to 9 AM
0	2019	54,564
1	2020	55,453
2	2021	56,357
3	2022	57,276
4	2023	58,210

More passes, however, should be performed in future works once more issues are resolved in order to garner more confidence in the results.

7.2 Population Scaling

According to the most recent census conducted in the Philippines in 2020, the Philippine population increased by 8,053,906 from 100,981,437 in 2015, translating to an annual population growth rate of 1.63% [15]. This growth rate shall be the basis of the population scaling scenarios conducted for the LRT-2. In these scenarios, the effects on the LRT-2 system of yearly increases in ridership volume, based on the said growth rate figure, will be examined through the train simulation. On January 28th in the year 2019, 54,564 trips were recorded between 4:00 and 9:00 AM in the LRT-2, as taken from the smart card data. Using the population growth rate figure along with this 2019 trip count, the yearly scaling up of the passenger ridership for the LRT-2 during the mentioned time period is projected from the year 2019 until the year 2023. Table 8 shows the projected ridership counts approximated until 2023 as computed using the given population growth rate percentage.

For each of the four future years to consider, three passes of the LRT-2 simulation will be executed with the same parameters as mentioned in Table 3. For this scenario test, the passenger spawn list is artificially scaled up from the January 28th, 2019 data to match the current simulation year's trip counts. In generating these additional trips, it is ensured that the likelihood of the trips to be generated will be based on empirical distributions.

Four metrics are of interest in the results for this scenario testing:

- (1) The *average travel time* of the passenger, defined as the time between tapping in at the entry station and tapping out at the destination station,
- (2) The *average time spent in the train system* by the passenger, also referred to as the *average time spent alive*, is defined as the time between spawning, possibly at the station entrance backlogs, and despawning at the exits of the destination station,
- (3) The *average load factors* of the trains throughout the simulation period, defined as the ratio of the number of passengers riding the train and its actual capacity, and

- (4) The *average platform waiting time* of the trains, which is how long the trains stop at the platforms to allow passengers to board and alight.

The first two metrics primarily deal with attributes relating to passenger crowds, while the last two are of the trains. This allows for the analysis of the effects of the behaviors of the trains on the passenger crowds, and vice versa. Figures 8, 9, 10, and 11 show scatter plots of these metrics aggregated across the three simulation passes for each of the projected years from 2020 to 2023.



Figure 8: A scatter plot showing the simulated mean passenger travel times from 2020 through 2023.



Figure 9: A scatter plot showing the simulated mean passenger times spent in the train system from 2020 through 2023.

As seen in Figure 8, it was observed that as estimated ridership increases through the years, increases in the mean travel times were also noticed. From an average travel time of 18.49 minutes in the 2019 simulation with around 54,000 passengers, it increased to 18.64 minutes in the 2023 projections with over 58,000 passengers, representing a 0.78% increase. For the mean time spent alive seen in Figure 9, increases were also observed as ridership scales up. In the 2019 simulation, passengers spent an average of around 29.1 minutes in the simulation. In the 2023 projections, this increases to over 29.73 minutes, representing a 2.16% increase. It can be surmised

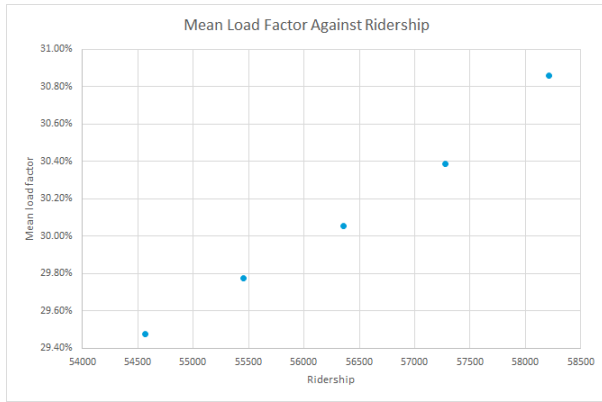


Figure 10: A scatter plot showing the simulated mean train load factors from 2020 through 2023.

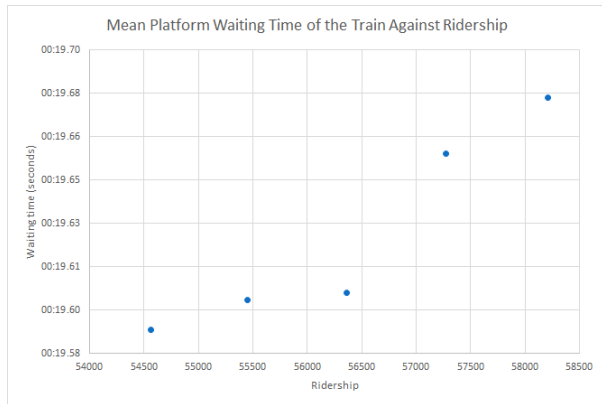


Figure 11: A scatter plot showing the simulated mean platform waiting times of the trains from 2020 through 2023.

from these observations that increases in the LRT-2 ridership result in slight increases in the travel times as well as the time spent by the passengers in the system itself.

As for the metrics related to train operations, as observed in Figure 10, the average load factor of the LRT-2 trains increases from 29.47% to 30.86% given the same increases in ridership, representing a 4.7% increase. Finally, platform waiting times of the LRT-2 trains (Figure 11) also saw increases in magnitude to 19.68 seconds, albeit with a difference of just around one tenth of a second from the 2019 mean platform waiting time of 19.59 seconds, representing a 0.47% increase. From these observations, it may be said that an increase in LRT-2 ridership also affects train operations as described by the previous metrics, but more so with the average load factors than with the average platform waiting times.

As mentioned previously, more passes and projections into the future should be performed in future works.

8 CONCLUSION

The train simulation was constructed with the comprehensive integration of both train operations and passenger crowd dynamics

in one model in mind. A detailed passenger movement framework based on the seminal social force model by [8] was incorporated into the passenger agent. To serve the twofold purposes of creating spatial station environments for the train simulation and testing how passenger crowds behave in such environments, the station editor was developed as a companion to the train simulation model.

The train simulation was validated against empirical data taken from real-world train systems. One of these empirical data were smart card tap-in and tap-out data, used as a reference for the travel times of the passengers in the developed model. The other class of empirical data used were video recordings of the train stations, used to qualitatively validate the movement of the passengers which were otherwise not captured in the ticket data. The face validation of the crowd dynamics showed that the relevant crowd simulation components and frameworks of the ABM capture most behaviors seen in the real-world. Meanwhile, quantitative validation was performed on the train system models with the smart card data from 4:00 to 9:00 AM. After analyzing the validation results as well as the output logs of the models, amendments were made to resolve some discovered issues. After such encountered problems were remedied, the models were revalidated in another iteration, and the new results examined, then compared and contrasted with those of the previous iteration. All in all, three iterations of resolutions and improvements were made to the models, resulting in models which captured train system dynamics better than the first iteration. Metrics showed that the LRT-2 model captured its real-world counterpart the best, relative to the other models.

Hypothetical scenarios were tested on the LRT-2 model. For instance, a varying number of trains was deployed to see how it would affect the passenger crowd volume. It was discovered that, for the time period of 4:00 to 9:00 AM, having less than 4 trains deployed results in a tipping point which represents a considerable increase in passenger travel times. Another scenario examined how increases in ridership with respect to the growth of the Philippine population affect the operations of the LRT-2 for four years after the empirical smart card data was recorded. Through simulations on an artificially scaled up version of the empirical data taken in 2019, projections were made for metrics which describe the performances of train operations and passenger crowd dynamics in the LRT-2. Increases of 0.78% in the mean passenger travel time, 2.16% in the mean total time spent in the system, 4.7% in the mean train load factor, and 0.47% in the time trains stop at the platforms were projected between 2019 and 2023.

In both the validation and scenario testing phases, technical limitations affected the rate at which the simulations were run and prevented more passes and parameters sweeps from being performed. In the case of the LRT-1 model, the simulation could only be run until a certain time after which the simulation was then seen to exhaust the requirements of the hardware which the simulation runs on. Hence, it is imperative that more optimizations and interventions be done on the developed models to resolve such technical and logistical issues.

We were able to develop an ABM that simulates train operations alongside passenger crowd dynamics simultaneously, and was able to support an integrated simulation of both trains and crowds in the context of Metro Manila's RTS infrastructure. However, refinements

to the model are needed in order to better capture real-world dynamics, especially for the LRT-1 and MRT-3 systems. Furthermore, due to such level of integration, performance optimizations have to be done in order to make the simulation scale better for subjects with larger scale requirements, such as in more comprehensive urban scenarios with more complicated behaviors and phenomena.

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